

GEOS B
SHIP POSITIONING USING SATELLITE RANGE DATA
ANALYSIS OF ERROR PROPAGATION AND
VISIBILITY PROBABILITY

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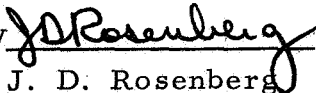
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National Aeronautics and Space Administration
GEOS B Project

Approved by 
J. D. Rosenberg
GEOS-B Project Manager

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Prepared for
National Aeronautics and Space Administration
GEOS-B Project


by

GEONAUTICS, Inc.
4626 Wisconsin Avenue, N. W.
Washington, D. C.

Under
Contract NASW 1247

Prepared by 
Dr. F. Calabria

Reviewed by 
E. E. Markson

Approved by 
Dr. A. Silvestrini

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SECTION 1

SUMMARY

1.1 OBJECTIVE

The technical objective of this report is to present a preliminary evaluation of the position determination of the Apollo tracking ships by using pure ranging techniques.

1.2 CONCLUSIONS

The navigation requirements of the Apollo ship program may be provided by Satellite Ranging techniques, combined with the Ship Inertial Navigation System. Ranging with the shipboard C-band radar on an accurately tracked satellite carrying a C-band transponder, such as GEOS B, will provide an independent source of navigational data. Based on the error analysis contained in this report, the navigational data provided by the C-band ranging technique should provide position data which will satisfy the position accuracy specification for the Apollo tracking ships 99.95% of the time. This information will be available on an average of 35 minutes per day, probably divided into three or four daily passes.

1.3 RECOMMENDATIONS

The analysis outline presented in this report should be expanded and a comprehensive systems analysis performed on the C-band ranging technique for ship positioning. Particular attention should be directed towards a comparison of system interface problems presented by the Doppler navigation technique and the C-band radar ranging techniques.

SECTION 2

DISCUSSION OF RANGE POSITIONING SYSTEM

This section describes some analytical parameters which characterize a C-band radar positioning system which might be implemented for the determination of ship position. The subjects discussed are the attainable accuracy of the C-band radar range measurements, the expected error in the satellite ephemeris, the Apollo program accuracy requirements for ship positioning, the probability of the satellite being visible during an average day on station, and finally a discussion of the accuracies achievable with the currently proposed inertial/Doppler system.

2.1 INTRODUCTION

The Manned Space Flight Network (MSFN) is an extensive complex of ground, sea and airborne stations which will play a vital role in the Apollo mission flights. This network must provide precision tracking data during all flight phases for the prediction of spacecraft position and for the development of navigation inputs to the guidance system.

The limiting errors in the global network are, for the most part, external to the tracking instrumentation themselves--arising from survey uncertainties, refraction uncertainties and others. Certain systematic errors arising from physical sources within the equipment also contribute significantly to the tracking system errors. Random noise is the smallest error contribution with state-of-the-art statistical filtering procedures. Indications are, at the present time, that the largest errors in the accuracy of the tracking system are due to uncertainties in geometric and gravimetric parameters (reference 1).

This report addresses itself to the positioning of one subset of the MSFN--the five tracking ships which are planned for use on the lunar missions to increase coverage. Under the auspices of the Apollo program, five ships are being converted and instrumented to provide coverage for the injection, insertion and reentry phases of the lunar flight. The coverage obtained with the ships is critical and provides a support capability which will not exist with land stations.

A typical position distribution and required on-station times for the five ships is given below in table 2-1. The ships are seen to be primarily in the equatorial latitudes and fairly widely dispersed in longitude. The most significant column in this table, however, is the required on-station time because of its resultant effect on the position accuracy problem.

TABLE 2-1

TYPICAL SHIP DISTRIBUTION FOR THE LUNAR MISSION

<u>Ship #</u>	<u>Position</u>		<u>On-Station Time</u>	<u>Mission Support Coverage</u>
1	23°N	47°W	14 days	Insertion
2	5°N	177°E	14 days	Injection
3	20°S	36°E	14 days	Injection
4	18°N to 12°S	170°W to 178°E	23 days	Re-Entry
5	16°N to 10°S	165°E to 178°E	23 days	Re-Entry

The Apollo program has imposed a requirement on positioning accuracy which dictates that the probable error in ship position in a suitable coordinate system shall not exceed 300 meters. If position is to be determined at discrete times for the purpose of updating the inertial system, the probable error of the position fix will have to be less than 300 meters to allow for error growth between fixes. How much less depends on the rate of error growth and the fix time interval.

The primary means of position determination is the Ship Inertial Navigation System (SINS) installed aboard the ships. The SINS is a highly accurate inertial system in which gyro drift effects are reduced to minimum levels by sophisticated techniques. This is generally achieved (reference 2) by various combinations of case rotation, gyroscopic redundancy, electromagnetic or electrostatic bearing support, and by determination of, and analytical correction for, gyro drift. However, to obtain the ultimate in accuracy, such systems require elaborate and lengthy calibration procedures. Therefore, a reduction in performance is generally allowed to permit an easier calibration.

Systems of the SINS class currently have error growth rates of about 0.1 to 1.0 nautical miles per hour: the present state-of-the-art is near the lower portion of this interval (reference 2). The unavoidable platform gyroscopic drift will be corrected by sidereal observations. A star tracker, which can be operated either manually or automatically, is provided with the inertial system. The onboard computer retains a 60-star catalog, thus providing the reference for the computations which check the platform orientation.

Stellar angular arguments measured with respect to the local vertical define circles on the earth's surface. These circular loci are formed by the intersections of the conical surfaces defined by the stellar angles and the spheroidal surface of the earth. The intersections of two such circles define two points on the appropriate spheroidal surface. Two or more stellar fixes, done sequentially or in parallel, can thereby determine an unambiguous point on the earth. This information may be obtained by processing a data mix comprised of local vertical reference data from the inertial navigation system and measured stellar arguments. In the SINS system, the astrotracker is physically coupled with the inertial platform. Alignment is accurate to within 1 second of arc. For this and other purposes, a flexure monitor is installed aboard the ship.

Ship location obtained as the result of the processing of stellar vertical data can be compared to the position data determined inertially. In this manner, a suitable basis may be established for the correction of inertial position errors. By other techniques, it is possible to determine

and compensate for the possible velocity errors. Although the stellar/inertial configuration is appealing, it has a number of weaknesses. Included among these are the following:

1. The system is vulnerable to breakdown of the inertial system since it requires local vertical information from the inertial system. Thus, there is no redundancy (in the sense that the two systems are not independent) and there is no improvement in system reliability.

2. Stellar data are basically "fair weather" data. In case of bad weather, no data will be available for a fairly long and somewhat unpredictable period of time.

To by-pass the weaknesses cited above and to maximize the probability of ship position being within the required probable error, it is desirable to provide an independent external positioning capability. Such a capability, if truly independent, will significantly increase the overall reliability of the ships tracking data. The use of standard navigation techniques, such as LORAN C, which is based on range-difference methods, is anticipated for inertial system correction purposes when the ship is in a favorable area. However, it is doubtful that LORAN C can provide the accuracy necessary for the purpose (reference 2).

Doppler data derived from a Transit type satellite can be used to correct the inertial platform output. The Apollo tracking ships will be able to use the Transit satellite system as an independent positioning device. This is because a suitable receiver will be installed onboard within the year 1967. The principal problem areas with the Doppler/inertial configuration lie first, in the complex system interface requirements, and second, in certain computational similarities of the two systems. That is, both systems need reasonably good initial condition information to achieve a solution to the positioning problem.

Another technique for deriving ship positioning is by means of a pure ranging technique, such as ranging on an orbiting C-band transponder. If the position of the orbiting transponder is known with sufficient precision, as for example on a geodetic satellite, it is feasible to transmit predicts to the ships on station when needed. Since all Apollo tracking ships will

carry C-band radar and GEOS-B will carry a C-band transponder, the possibility of precise positioning of the Apollo tracking ships by pure ranging techniques is most appealing and the analytical procedure is developed in Appendix A of this report.

The present state-of-the-art in ranging techniques and orbit determination and prediction are discussed in this report. The accuracies presently attainable are reported. These accuracy data are necessary for a transformation of errors in range measurement and satellite position into ship position error as discussed in section 3 and Appendix B.

It is, of course, vitally important to know how often a fix can be obtained from the GEOS-B satellite if an inertial system, whose gyroscopic drift is known, is to be updated by satellite ranging. The probability of satellite visibility is evaluated as a first approximation in section 2.5.

In conclusion, it is emphasized that the present report is primarily an outline of an error analysis of a ship positioning system using range data. However, the order of magnitude of attainable precision can be obtained from the data reported in the discussion to follow.

2.2 RANGE MEASUREMENT ACCURACY

Range measurements taken with a calibrated radar unit and corrected for the hypothesized physical environment will generally consist of the sum of two range components: the true value for range and the error in the measured range. The measurement error may consist of a random component and one or more systematic components which are often of a complex, interdependent, nature. If several repeated measurements show that the variation of the measurement error is small--as estimated by the standard deviation--then the range measurement is considered to be precise. Accuracy, though, is a function of both random and systematic components. Thus, if systematic errors are high but standard deviation is very low, then the associated range measurement is inaccurate but very precise.

As an example, (reference 1), if we are trying to measure a fundamental constant such as the velocity of light in vacuo, then we are, of course vitally interested in both (1) the precision of measurement, and (2) the bias or systematic errors of the measuring technique or instrument, which we would hope to take care of by proper calibration and analysis. The bias may be referred to also as the "constant systematic error" for a single test, since for a given experimental set-up it may remain constant for a series of measurements on a particular occasion or day. It might be expected that ten measurements of the velocity of light on a given occasion or day, therefore, would consist of a constant bias or systematic error and ten random errors of measurement. On the other hand, measurement of the velocity of light on a different day or occasion -- even with the same instrumental set-up -- may result in a different bias or systematic error from occasion to occasion, which unfortunately may not be predictable. Indeed, such is precisely the case with measurements of physical constants and many other fundamental quantities, and also with measurements made on missiles in flight. Moreover, it is very often true that the variation in systematic errors is unaccountably large, being frequently many times greater than the random error of an individual measurement. The systematic errors, therefore, may generally be (and do indeed turn out to be) the most important source of inaccuracy for many tracking systems.

The total variance in the errors of measurement may be broken down into several key components. It is generally possible to estimate the standard error of any single, original measurement of a tracking system. Part of the random error will be high frequency "noise" of purely random character and another part will be of a low frequency cyclical character, requiring spectral analyses. Then, fixed biases and systematic error trends together with the random errors and low frequency variations may be assessed in size by the standard deviation.

Factors contributing to instrument errors are bias, scale factors, timing, transponder and receiver jitter, cycle jump counts, etc. Dimensional transformation errors have to do with scaling of raw measurements (such as phase in a CW system) into a measurement coordinate format, such as range or range difference, required for subsequent analyses. These errors would also include physical uncertainties such as refraction errors, position errors

and speed of light uncertainty. Finally, data processing and mathematical reduction "errors" would include the various coordinate rotations, smoothing, differentiation and other analytical manipulations. No amount of effort devoted to improving system hardware by reducing instrument errors can affect the latter two sources of error; (i. e., data processing and mathematical reduction) and improvements in the latter two sources cannot themselves improve the former.

The tropospheric range measurement bias error will depend principally on the different propagation velocities in a vacuum and in the atmosphere. In addition to that, a second order effect is present, due to the difference between the actual ray path length and the line-of-sight path length (minimum distance line). Quantitative data are available from reference 3 for the C-band propagation tropospheric biases versus the antenna elevation angle. If no correction is attempted, the range bias is about 24m at 5° elevation angle and 13 m at 10° , slowly decreasing as the elevation increases to become about 4m at 30° . However, these biases can be greatly reduced by partial compensations. A simple correction based on a standard atmosphere reduces this error to about 0.5 m for elevation angles above 5° .

Short term range fluctuations with periods up to ten minutes are also present, primarily depending on the cloud cover conditions. This effect increases as the cloud cover increases, a heavy cumulus cover creates the worst conditions with an estimated error of $\pm .5$ m at elevation of 5° and ± 0.25 m at 20° . Both errors are at the 1 σ level.

The GEOS B satellite is at the upper limit of the ionosphere, whose interposition causes a bias error by increasing the time delay relative to vacuum propagation. As stated, this systematic error is approximately inversely proportional to the frequency squared. It also depends on the ionosphere conditions, being larger during daytime and in conditions of extreme atmospheric disturbance such as occur during abnormal solar activity. Data are available (reference 4) which show that at C-band frequencies average ionosphere and antenna elevation angle of 5° the bias is 0.5 m at nighttime and 1.5 m at daytime. This error also decreases as the elevation angle increases. In conditions of extreme atmospheric disturbance, the bias error can be up to 5 m. Another sizable source

of error is the present uncertainty in the knowledge of the velocity of wave propagation. The velocity of light is at present known with the precision of one part in 10^6 . This causes, even at the modest ranges of the GEOS Satellite, an error up to 1.5 m.

Overall C-band radar range accuracy data are obtainable from reference 5 which describes a simulation performed for the Apollo project. In this case a 1σ range noise of ± 9 m and range bias of ± 18 m were assumed for ship-borne C-band radars. These values are presumed after correcting for ship motion, speed, local vertical and refraction. These data are believed to be consistent with those previously presented in this paragraph.

2.3 SATELLITE POSITION ACCURACY

The uncertainty in the satellite location in space at a specified instant is one of the most important factors controlling the ship positioning precision. The accuracy attainable in determining satellite position coordinates is strongly related to orbit determination and orbit prediction problems. A review of the methods presently used for the GEOS A satellite gives an estimate of what can be expected from GEOS B

The GEOS satellite orbit determination is based on data from the NASA Minitrack network. The Minitrack system uses an interferometer technique to provide angular data whose error is on the order of 0.2 to 0.3 milliradians. Since tracking data are normally obtained at elevation angles in excess of 45° , the GEOS B position error from orbit determination can be estimated to be about 0.3 km when no improvement is made in the orbit determination process.

This accuracy will degrade with time when orbit prediction is attempted. Deterioration of precision is ascribed both to error in the initial conditions and to imperfect knowledge of the orbit perturbing factors. Experimental data show that the GEOS A satellite orbital data show in general, a disagreement with the predicts of 0.25 degree after a one-week period. In units of time the error is about 1 second after the same time frame. Because of the fact that the GEOS A will also provide a better knowledge of the earth gravity field, the last figures may be improved for the GEOS B satellite if these improvements are incorporated in the prediction model.

Better precision is achieved from the GEOS A history tapes. These tapes are generated for gravity model studies which will eventually produce refinements to the gravity field. This is a dynamic mode to recover geodetic parameters using theory of satellite perturbations. The satellite position error in the orbit is estimated to be from 0.1 to 0.2 km. This error would represent the best accuracy attainable using data reduction programs currently in operation.

2.4 SHIP POSITION ACCURACY REQUIREMENTS

The ships involved in the Apollo program use inertial techniques as the primary method for position determination. The degree of success achievable by these methods depends on the allowable positional errors, the accuracy and availability of external position information for periodic checks of the inertial platform outputs, and, of course, on system reliability.

Inertial navigation equipment has the property of high accuracy for short time span. In addition, continuous positional information is available from an inertial system. A further advantage of the system is that it is self-contained, thus improving, under equivalent conditions, the reliability.

An inertial system, however, is subject to a drift, even in the absence of vehicle motion. The accuracy of position information degrades with time so that periodic corrections, based on methods with adequate accuracies, are required if the position accuracy is to be maintained.

Since the primary system is self-contained, its reliability does not depend upon external factors. The updating system, however, is affected by external factors and therefore a redundant position determination method is required for reliability.

Accuracy is generally accepted as a measure of navigation system performance. The choice of an updating system for inertial platform correction depends strongly on the required ship position accuracy. External positional information can be obtained via radio techniques when available, landmarks, ships with known positions, celestially derived information, including satellites, or some combination of these methods. System accuracy must be intended in its broadest sense to include the effects of the equipment, accuracy of the information regarding initial conditions, set-up procedures and

vehicle behavior.

A simulation concerning the navigation capabilities for the Apollo project (reference 5) yields data on ship position accuracy requirements. In this study, the ship location was $21^{\circ} 15'$ North latitude and $48^{\circ} 45'$ West longitude with a bias error (at the 1 σ level) of ± 540 m in both N-S and E-W directions. This corresponds to a Probable Error of about 360 m. The present position accuracy requirement for the Apollo program tracking ships is a Probable Error of 300 m. This figure is adopted in the present study.

2.5 SATELLITE VISIBILITY

The problem of satellite visibility is a vital one if the GEOS B satellite is to be used for position fix purposes. If the inertial platform drift and the desired accuracies are known, the maximum allowed time between corrections is fixed. The probability of satellite visibility is to be compatible with this figure and is defined as follows.

Each Apollo ship has the capability of sending signals to and receiving signals from the GEOS B satellite along line-of-sight paths anywhere within a region above some minimum antenna elevation angle α and through azimuth angles of 0° to 360° . Thus, for each ship the paths to the satellite are contained within a conical region with an apex angle at the ship of $180^{\circ} - 2\alpha$. Each cone may be visualized as intersecting the sphere at satellite orbit altitude, thereby forming a circular boundary wherein the satellite is in a useful position for navigation purposes. This region is called circle of view from the ship. The satellite visibility probability is defined as:

$$P_1 = t/T$$

where t is the time the satellite is within the circle of view during a long period of time T .

This is the definition of probability that a satellite pass will be useful for navigation purposes. Another satellite visibility probability can be:

$$P_2 = n/N$$

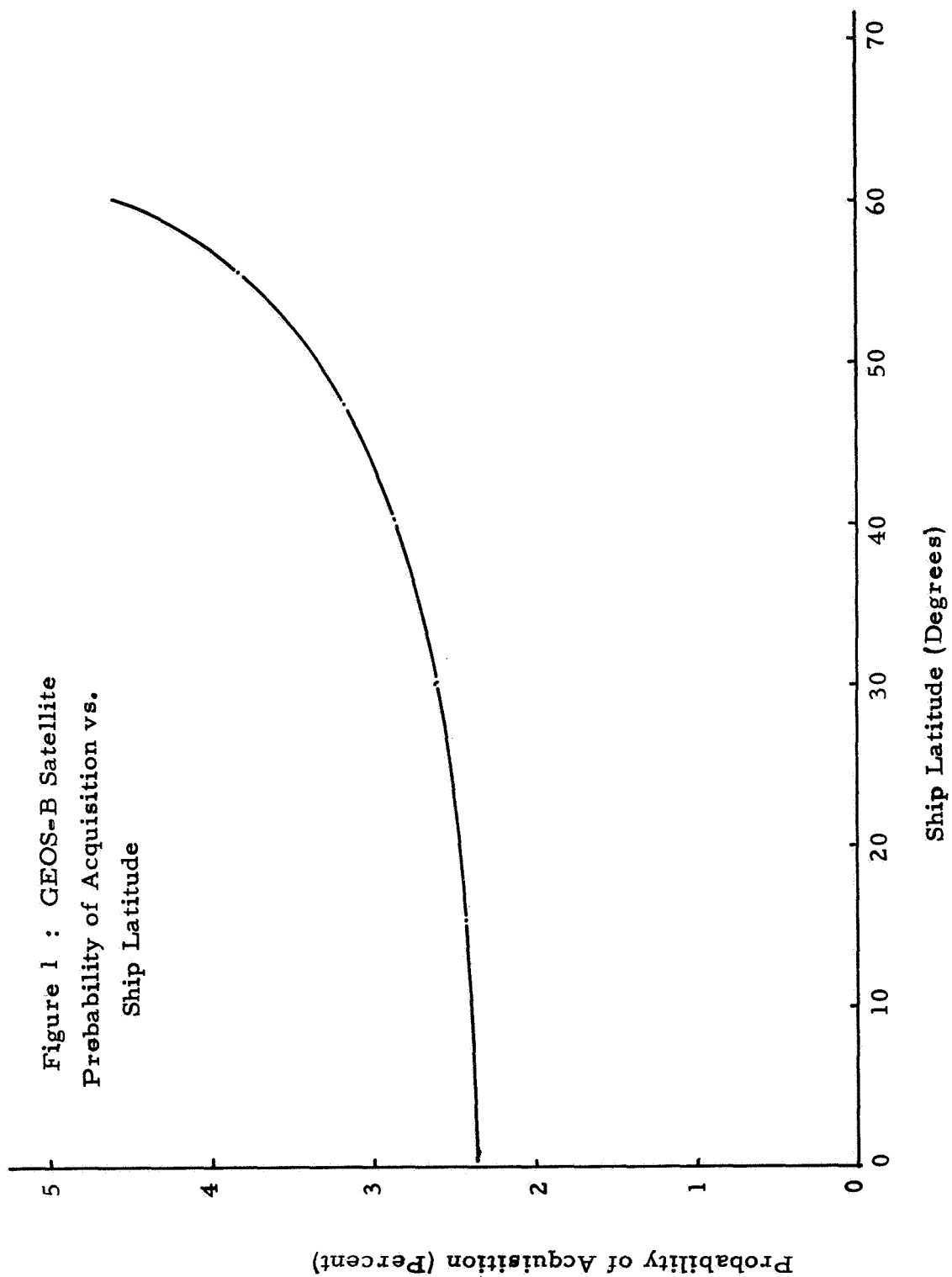
where n is the number of times the satellite orbit intersected the circle of view during a large number of satellite revolutions N .

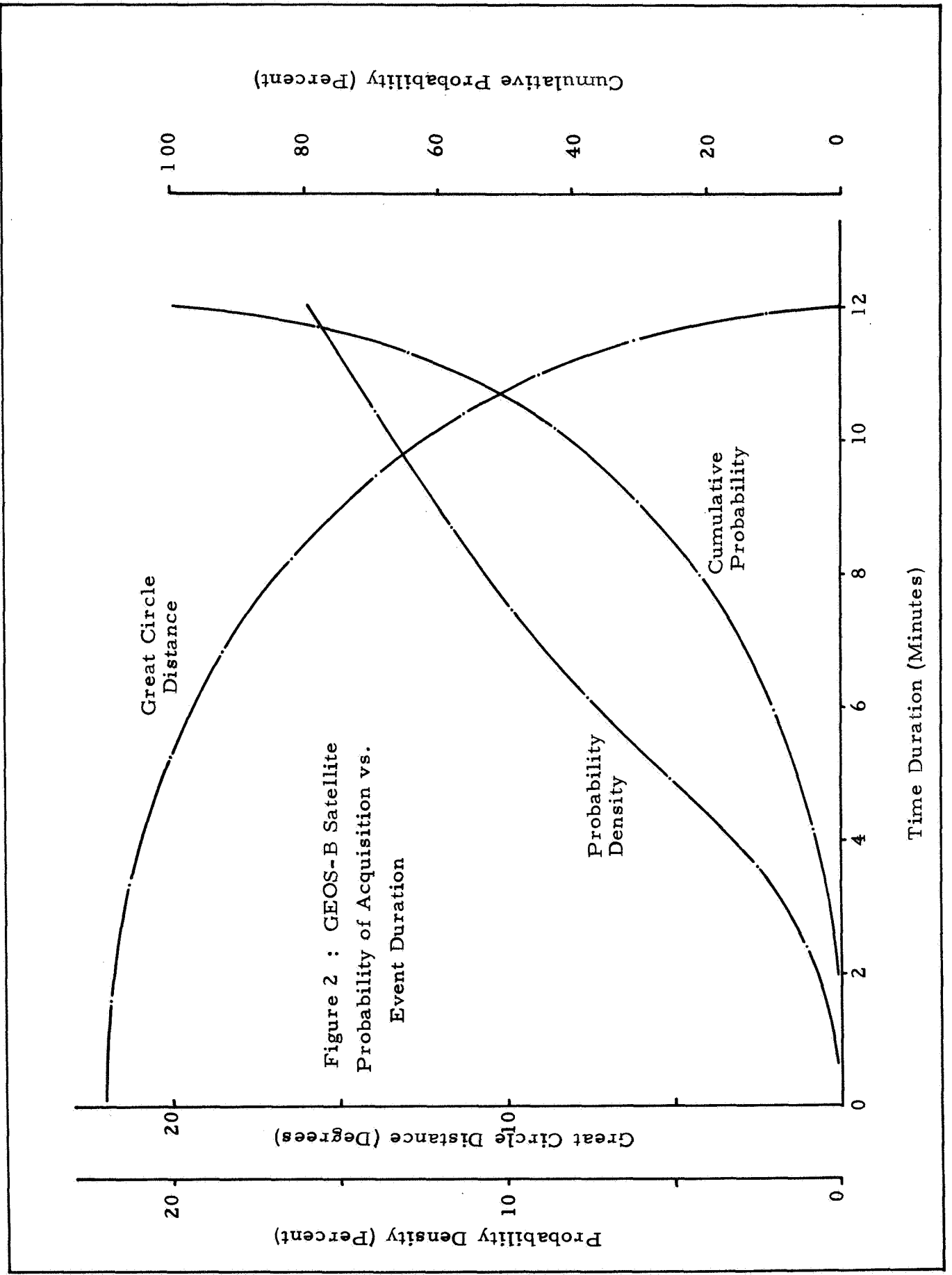
The probability of satellite visibility, P_1 , as defined in reference 6 and 7, is the percentage of time the satellite is in a position useful for navigational purposes. A study is presented in reference 7 for navigation satellites (i. e., circular regions of mutual communication). However, the study does not extend to altitudes as low as that of GEOS, presumably because they are considered to be non-optimum for navigational purposes. Taking advantage of the fact that the satellite orbit is 74° inclination, approaching polar, a rapid evaluation of visibility probability (reference 8) is possible. The results are shown in figure 1. They are valid for polar orbit, satellite altitude 1000 km and minimum elevation angle of 10° . These results are slightly conservative in the equatorial regions because these regions are better served if the orbit inclination decreases. Up to 30° latitude, north or south, the probability is approximately constant at about 2.4%. With an orbital period of about 111 minutes, the satellite will be within view of a ship in the afore said equatorial strip 34.7 minutes per day average or 2.7 minutes per orbit average.

Since the satellite is to be used for navigation purposes, this information is useful but not complete. In order to perform range measurements, the satellite pass duration must be more than some minimum acceptable limit. On the other hand, if the pass is overhead (maximum duration), very poor position information is obtained in the direction perpendicular to the orbital plane. In a previous study (reference 9) based on synchronous satellites, it is shown that ship positioning is acceptable when the great circle distance between the ship and the orbital plane exceeds 5 degrees. Therefore, an upper limit on pass duration also exists and is defined by the minimum great circle distance between the ship and the orbital plane.

For the purpose of selecting really useful passes, the diagram of figure 2 was derived. It relates pass duration to the cumulative probability that the pass will have that duration or less and to the great circle distance of the ship from the satellite orbital plane. The diagram is valid with fair accuracy for ship latitudes up to $\pm 30^\circ$. Using the cumulative probability curve of the diagram, the estimate of the percentage of useful passes is simple and

Figure 1 : GEOS-B Satellite
Probability of Acquisition vs.
Ship Latitude





straightforward. For example, if the minimum time duration of a pass is stated to be four minutes and no maximum duration time is specified, then 4% of the passes are not useful. If in addition a minimum great circle distance of 5 degrees is to be required (in accordance with the results of reference 9), an additional 23% of the passes must be discarded. Therefore, only 73% of all passes through the ship circle of view are useful for position determination.

The probability P_2 gives the expected percentage of in-view passes regardless of their time duration. In analogy with P_1 , P_2 is minimum at the equator and fairly constant up to $\pm 30^\circ$ latitude. When the ship is at the equator $P_2 = .25$ and therefore, considering a great number of orbits, one orbit in every four will intersect the circle of view. Combining together the results obtained in this section it can be said that, if an average is made over a long period of time, the GEOS-B satellite will be in view for 35 minutes per day. This time will probably be divided into three or four daily passes (3.2 passes per day average).

SECTION 3

COMPARISON WITH RANGE-RATE TECHNIQUES

3.1 RANGE-RATE TECHNIQUE ACCURACY

The range rate technique for ship positioning involves measurement of the rate of change of the distance from the satellite to the ship during a pass. This is generally done in an indirect way utilizing the Doppler effect resulting from the motion of the satellite relative to the observer.

For maximum accuracy, all available information is to be extracted from the phenomenon and therefore the measurements are continued through the entire pass of the satellite. Data are obtained only when the satellite is in view, and are not available at all times as in systems such as Loran. Therefore, checks of previous computations can only be performed after a sizable interval of time.

Error can be reduced by processing all the data instead of only those within the interval of time immediately adjacent to the null Doppler shift. The principal error sources are uncertainties in satellite position, error in the frequency standard, propagation effects and uncompensated motions of the receiving antenna. These errors can be amplified by unfavorable problem geometries. Zenithal and low elevation angle satellite passes provide positional information but do not result in a satisfactory position fix. The Doppler positioning system introduces a position ambiguity. Two station positions are theoretically possible, specularly symmetric to the subsatellite point trail on the surface of the earth. This problem is easily solved with sufficient data and good geometry.

If great care is taken in data processing, the positioning precision attainable using Doppler systems is remarkable. For example, 26 observations were made from a station at a known location (reference 10) giving the results shown in the table below.

TABLE 3-1

<u>Number of times</u>	<u>Error in meters</u>
10	30 to 800
9	80 to 160
6	160 to 800
1	800 to 1050

However, shipboard data processing is expected to be of lower quality than that used for this example, and, accordingly, the credibility of the results degraded.

As report on the results obtained by the U.S. Navy (reference 11) from Doppler observations for geodetic purposes claims that 25 meters accuracy globally is not completely optimistic and estimates that the Doppler global station determinations at this time are probably good to within 40 meters. However, it takes considerable effort to obtain such accuracies in position. These solutions take into account data collected from several primary stations globally distributed and held fixed in the solution. Therefore, considerable data filtering and aggregation time as well as many hours of computer processing for the batch adjustment were required. This situation will not exist when the system is used for ship positioning.

Dual frequency method of correcting ionospheric errors is standard practice, although agreement is not reached on the frequencies to be adopted. Frequencies of 162 and 324 Mc/s are customary for most Tranet stations, while 324 and 972 Mc/s have been suggested by APL for use during periods of high solar activity.

The present minimum of solar activity probably facilitates the ionospheric error correction. However, the sunspot activity is continuously increasing. A maximum will be reached in 1971 that will probably degrade the Doppler data reliability to some extent.

A C-band characteristic is its relative immunity to ionospheric errors. Therefore, the use of C-band frequencies is appealing, since the

Apollo program will take place near the peak of activity in the solar cycle.

3.2 RANGE TECHNIQUE ACCURACY

The accuracy of a ranging technique in which the GEOS B satellite participates directly in the establishment of Spherical Surfaces of Position (SSOP) is investigated in Appendix B. Three SSOP's intersect in a point (or at most, two points), and in the system selected for detailed analysis, two of the three SSOP's are formed by measuring the range from the satellite at different times; the third SSOP is formed by the estimation of the ship distance from the earth's center.

A pseudo-astronomical coordinate system, consisting of declination and right ascension angles with respect to the satellite orbital plane, is adopted in the actual approach to the error analysis. This greatly simplifies the analytical work to be accomplished, but results in an incomplete error model. Errors perpendicular to the orbital plane are not taken into account, thus producing optimistic error analysis outputs. The addition of this error source causes the error level to increase by an amount which can be evaluated fairly well. One can prove that the system accuracy is still largely adequate, once all possible errors are taken into account.

Appendix B deals with an error propagation analysis applied to a typical ship/satellite geometry. The output of this analysis is the ship position uncertainty on the spheroid surface. Both systematic and random error propagation are investigated and the results are compared with the Apollo mission requirements as stated in section 2.4. The resultant Probable Error (P. E.) in ship position on the spheroid is about 91 m. The probability of the hypothesis $P.E. > 300 \text{ m}$ is .05%. There is, therefore, a "statistic certitude" that the system will satisfy the Apollo mission requirements. Thus, the analysis shows that the Apollo requirements are met 99.95% of the time.

The analysis presumes error levels for the measured quantities that are consistent with operational systems. These assumptions are probably conservative when projected in the GEOS-B time frame.

The error analysis shows that the errors produced by C-band radar range inaccuracies are small compared with errors coming from other sources. In particular, the error produced by the imperfect knowledge of

the satellite position is prevalent. Improvements in ship positioning accuracy with the proposed ranging system is primarily a function of orbit prediction capability. The conservative assumption of a standard error in satellite position estimate $\sigma = \pm 100$ m is made in this error analysis as it is presently being achieved for GEOS-A. The GEOS-A satellite will allow a better knowledge of the perturbing forces (i. e. gravity parameters, etc.) acting on a satellite. Thus the accuracy of the GEOS-B ephemerides is expected to be increased. As an example of the improvement in accuracy which will result from improvement of the satellite ephemerides, consider a reduction in position uncertainty to 50 m from the assumed 100 m. This results in a ship position P. E. of 48 m--a nearly proportional improvement in accuracy.

Comparison with Range Rate techniques shows a Range technique to be potentially more accurate. Moreover, the Range Rate figures were obtained by a ground station, using sophisticated data processing techniques, while the present figures come from conservative assumptions with no endeavor to statistically ameliorate the final accuracy outputs by an averaging process.

The GEOS-B operation during a period of maximum solar activity causes a range positioning technique to be appealing. In fact, C-band frequencies are relatively immune to ionospheric disturbances. The current accuracy figures will remain practically unaltered.

One realizes that the results of the analysis are not complete. The method accuracy is adequate but its necessary availability has not been demonstrated. To be suitable as a primary platform correction technique, the satellite availability must be adequate to allow the ship to remain within the stated position accuracy limits. Thus, a study is recommended combining together position accuracy data, satellite availability probability data, and inertial platform drift data. Final output of the study should be a function relating the probability of remaining within the stated accuracy limits versus the time elapsed from the last platform correction.

Accuracy and availability data are reported in the present study. Unfortunately, precise inertial platform drift data are not presently available at the unclassified level. This last step must therefore remain beyond the goals of the present study.

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APPENDIX A

COMPUTATION OF SHIP POSITION FROM RANGE DATA

The ship position can be computed using methods requiring either three or four range data from points whose positions at the instants of measurement are known.

However, the three-range method is applicable only when the ship position is approximately known. Geometrically, a range datum from a known point defines a spherical surface of position centered at the known point. The actual ship position is on the boundary of this sphere. Because the intersection of three spheres, if it exists, generally consists of two points, a means of solving the ambiguity is needed.

If the computation of ship position is made in order to carry the position data derived from the inertial platform mounted onboard the ship, the three ranges system is theoretically possible because the inertial platform data will generally solve the ambiguity.

The practical applicability of the three ranges method depends in this case on accuracy consideration.

It is worth noting that the ranges need not all be from the satellite. It is very appealing, for instance, to use as a point of known position the center of the earth. The earth radius at the ship position can, in fact, be known to a good approximation. In this way, only two range measurements from the satellite need to be taken. An advantage of this procedure is that two range measurements can be taken from the same satellite pass, while three ranges from the same orbit lead to unfavorable problem geometry. The use of the earth's center as a control point will therefore lead to easier and quicker position determinations.

A four range position determination, three measurements from the satellite and the earth radius estimate, is generally more precise and does not need previous knowledge of the ship position to solve the ambiguity. However, all three range measurements should not be made during the same satellite pass because it leads to poor geometry for position determination. Therefore, a second pass is needed that will happen in the most favorable case after

111 minutes for the GEOS B satellite. That will also involve considerable problems in the case that the ship is in motion. Dead reckoning, which often can be a source of sizable errors, is required to link together the observations.

For this Appendix, the Equations for position determination for both the three range and four range concepts are derived. The reference frame is in both cases earth centered with the z-axis along the axis of rotation of the earth, positive toward North and the x-axis toward the longitude origin. The equations are derived with all known points at generalized locations. If one of them is assumed to be the center of the earth the equations can be easily modified, and are, in fact, simplified.

Definitions and General Equations

For the sections that follow, the ship position will be denoted by P and its coordinates by x, y, z. The known points will be denoted by P_i ($i = 1, 2, \dots$) and their coordinates by x_i, y_i, z_i .

Therefore, the typical basic equation of the problem is:

$$R_i^2 = (x-x_i)^2 + (y-y_i)^2 + (z-z_i)^2 \quad (1)$$

where R_i denoted the range from the point P to the point P_i . A typical difference equation that can be derived from (1) is:

$$\begin{aligned} R_i^2 - R_j^2 = & 2 (x_j - x_i) x + 2 (y_j - y_i) y + 2 (z_j - z_i) z + \\ & + x_i^2 - x_j^2 + y_i^2 - y_j^2 + z_i^2 - z_j^2 \end{aligned} \quad (2)$$

Equation 2 can be more concisely written as:

$$K_{ij} = (x_j - x_i) x + (y_j - y_i) y + (z_j - z_i) z \quad (3)$$

where:

$$K_{ij} = 0.5 (R_i^2 - R_j^2 + x_j^2 - x_i^2 + y_j^2 - y_i^2 + z_j^2 - z_i^2) \quad (4)$$

Therefore, the K_{ij} are known parameters of the problem.

Three Range Position Determination

To simplify the problem, an auxiliary coordinate system is adopted. Once the coordinates of P are found with respect to this axis frame, they will be referred to the original frame by means of coordinate rotations and translations. The auxiliary rectangular axis frame, a, b, c, is such that the ab plane contains all three known points, the origin of axis is at point P₁ and the b axis is pointed toward P₂. If a, b, c, are the coordinates of P in this frame, the following relationships hold:

$$\begin{cases} R_1^2 = a^2 + b^2 + c^2 \\ R_2^2 = a^2 + (b-b_2)^2 + c^2 \\ R_3^2 = (a-a_3)^2 + (b-b_3)^2 + c^2 \end{cases} \quad (5)$$

Rearranging:

$$\begin{cases} R_2^2 - R_1^2 = -2b_2b + b^2 \\ R_3^2 - R_1^2 = -2a_3a - 2b_3b + a_3^2 + b_3^2 \end{cases} \quad (6)$$

Or better:

$$\begin{cases} b_2b = K_{12} = 0.5 (b_2^2 - R_2^2 + R_1^2) \\ a_3a + b_3b = K_{13} = 0.5 (a_3^2 + b_3^2 - R_3^2 + R_1^2) \end{cases} \quad (7)$$

Then, from Equation (7):

$$\begin{cases} b = K_{12}/b_2 \\ a = (b_2K_{13} - b_3K_{12})/b_2a_3 \end{cases} \quad (8)$$

Substituting Equation (8) into Equation (5):

$$c = \sqrt{R_1^2 - \frac{K_{12}^2}{b_2^2} - \left[\frac{b_2 K_{13} - b_3 K_{12}}{a_3 b_2} \right]^2} \quad (9)$$

The value of a_j, b_j ($j = 2, 3$) can be found from the values of x_j, y_j, z_j .

$$\begin{cases} u_j = x_j - x_1 \\ v_j = y_j - y_1 \\ w_j = z_j - z_1 \end{cases} \quad j = 2, 3 \quad (10)$$

$$A = \arctan \frac{u_2}{v_2} \quad (11)$$

$$\begin{cases} p_j = u_j \cos A - v_j \sin A \\ q_j = u_j \sin A + v_j \cos A \\ r_j = w_j \end{cases} \quad (12)$$

$$B = \arctan \frac{r_2}{q_2} \quad (13)$$

$$\begin{cases} l_j = p_j \\ m_j = q_j \cos B + r_j \sin B \\ n_j = -q_j \sin B + r_j \cos B \end{cases} \quad (14)$$

$$C = \arctan \frac{n_3}{l_3} \quad (15)$$

$$\begin{cases} a_j = l_j \cos C + n_j \sin C \\ b_j = m_j \end{cases} \quad (16)$$

Having calculated a, b, c , the corresponding values of x, y, z can be found as follows:

$$\begin{cases} l = a \cos C - c \sin C \\ m = b \\ n = a \sin C + c \cos C \end{cases} \quad (17)$$

$$\begin{cases} p = l \\ q = m \cos B - n \sin B \\ r = m \sin B + n \cos B \end{cases} \quad (18)$$

$$\begin{cases} u = p \cos A + q \sin A \\ v = -p \sin A + q \cos A \\ w = r \end{cases} \quad (19)$$

$$\begin{cases} x = y + x_1 \\ y = v + y_1 \\ z = w + z_1 \end{cases} \quad (20)$$

It can be seen that the ambiguity in ship position determination is introduced by Equation (9). The two possible solutions give the two possible positions of the ship.

Four Range Position Determination

The difference equations are, as before:

$$K_{L1} = (x_1 - x_L) x + (y_1 - y_L) y + (z_1 - z_L) z \quad (L = 2, 3, 4) \quad (21)$$

where

$$K_{L1} = 0.5 (R_L^2 - R_1^2 + x_1^2 - x_L^2 + y_1^2 - y_L^2 + z_1^2 - z_L^2) \quad (22)$$

The following definitions are made:

$$D = \begin{vmatrix} x_1 - x_2 & y_1 - y_2 & z_1 - z_2 \\ x_1 - x_3 & y_1 - y_3 & z_1 - z_3 \\ x_1 - x_4 & y_1 - y_4 & z_1 - z_4 \end{vmatrix} \quad (23)$$

$$N_x = \begin{vmatrix} K_{21} & y_1 - y_2 & z_1 - z_2 \\ K_{31} & y_1 - y_3 & z_1 - z_3 \\ K_{41} & y_1 - y_4 & z_1 - z_4 \end{vmatrix} \quad (24)$$

$$N_y = \begin{vmatrix} x_1 - x_2 & K_{21} & z_1 - z_2 \\ x_1 - x_3 & K_{31} & z_1 - z_3 \\ x_1 - x_4 & K_{41} & z_1 - z_4 \end{vmatrix} \quad (25)$$

$$\begin{array}{l} \text{Then:} \\ N_z = \end{array} \begin{vmatrix} x_1 - x_2 & y_1 - y_2 & K_{21} \\ x_1 - x_3 & y_1 - y_3 & K_{31} \\ x_1 - x_4 & y_1 - y_4 & K_{41} \end{vmatrix} \quad (26)$$

$$\left\{ \begin{array}{l} x \\ y \\ z \end{array} \right. = \begin{array}{l} \frac{N_x}{D} \\ \frac{N_y}{D} \\ \frac{N_z}{D} \end{array} \quad (27)$$

For this case, no coordinate transformations are required and no ambiguities arise.

APPENDIX B

RANGE SYSTEM ACCURACY EVALUATION

An error propagation outline is derived in this appendix for both the three range and four range cases. Numerical results are thus obtained, for the three range cases, by using a more convenient coordinate frame.

Observed problem parameters are:

$$\begin{array}{cccc} x_i & y_i & z_i & R_i \end{array} \quad \begin{array}{l} (i=1, 2, 3) \text{ (three range case)} \\ (i=1, 2, 3, 4) \text{ (four range case)} \end{array}$$

Both systematic and random errors will affect, in the most general case, these observed quantities. The propagation of these errors into uncertainties for the ship position coordinates (x, y, z) is the main goal of this appendix.

Error propagation formulae are based on the concept that the linear terms in Taylor's series can be used to express with sufficient accuracy the effect of small errors of the observed quantities.

Random, uncorrelated errors will therefore propagate according to the formula stating that the variance of the derived parameter is equal to the sum of the observed quantities' variances, each multiplied by the square of its own amplifying factor. This statement holds only for uncorrelated random errors.

Systematic errors will, on the converse, propagate by strict Taylor's expansion. Thus, the result can even be zero.

Rearranging the equations of Appendix A, they can be made explicit for x, y, z .

$$x = F_1(x_i, y_i, z_i, R_i) \quad (1)$$

$$y = F_2(x_i, y_i, z_i, R_i)$$

$$z = F_3(x_i, y_i, z_i, R_i)$$

The random errors propagate as follows: (2)

$$v_x = \sum_{i=1}^N (\dot{F}_1)_{x_i}^2 \cdot v_{x_i} + \sum_{i=1}^N (\dot{F}_1)_{y_i}^2 \cdot v_{y_i} + \sum_{i=1}^N (\dot{F}_1)_{z_i}^2 \cdot v_{z_i} + \sum_{i=1}^N (\dot{F}_1)_{R_i}^2 \cdot v_{R_i}$$

Analogous relationships hold for y and z.

The symbol v denotes the variances; N can be = 3 (three range case) or = 4 (four range case); the symbol:

$$(\dot{F}_1)_{xi} = \frac{\partial F_1}{\partial xi}$$

Systematic errors, if sufficiently small, propagate according to the formula:

$$\Delta x = \sum_{i=1}^N (\dot{F}_1)_{xi} \cdot \Delta xi + \sum_{i=1}^N (\dot{F}_1)_{yi} \cdot \Delta yi + \sum_{i=1}^N (\dot{F}_1)_{zi} \cdot \Delta zi + \sum_{i=1}^N (\dot{F}_1)_{R_i} \cdot \Delta Ri \quad (3)$$

Again, analogous relationships hold for y and z.

The error analysis problem may therefore be approached in a straight forward way both in the three and in the four range cases. The analytical work of deriving the explicit functions and obtaining partial derivative expressions can be, on the converse, too long for the limited goals of the present paper.

A declination/right ascension coordinate system with respect to the orbital plane can noticeably simplify the error analysis in the three range case still yielding useful results. This approach will be adopted in the present analysis, bearing in mind that the simplification is paid by an incompleteness in the error model.

The error analysis that follows evaluates the error in determining position with respect to the actual orbit plane. To go from this to a determination of position in a more conventional (geographic) coordinate system requires knowledge of two additional orbit parameters (for example, orbit inclination and ascending node longitude) at the time of measurement. These are, of course, in error and will contribute to the overall position uncertainty. The errors not taken into account are those associated with the uncertainty in satellite position perpendicularly to the orbital plane. It is nevertheless believed that the adequacy of the system for Apollo ship positioning purposes is demonstrated. This is inferred because:

- a) This error has at most the same value that the other two satellite position errors introduced in the analysis.
- b) Its effect on the ship position error will be, because of problem

symmetry, approximately of the same order of magnitude as the other errors introduced by satellite position errors.

c) The accuracy results from the analysis show a margin of safety such that the system is still adequate after the new errors' introduction.

An evaluation of the accuracy of a three range system (two ranges from a satellite pass and the distance from the ship to the center of the earth) can be achieved more easily, using a more convenient coordinate frame. If the coordinate μ is the anomaly in the orbital plane of the ship-point, computed from a convenient origin and ν is the great circle distance of the ship-point from the orbital plane, the following relationships (Reference 9) hold:

$$\nu = \arccos \left\{ \left[k_1^2 - 2k_1k_2 \cos(\mu_2 - \mu_1) + k_2^2 \right] / \left[2aR \sin(\mu_2 - \mu_1) \right]^2 \right\}^{1/2} \quad (4)$$

$$\mu = \arctan \left\{ \left[k_1 \cos \mu_2 - k_2 \cos \mu_1 \right] / \left[k_2 \sin \mu_1 - k_1 \sin \mu_2 \right] \right\} \quad (5)$$

Equations (4) and (5) give the coordinates of the ship in the present reference frame. When:

$$k_1 = a^2 + R^2 - R_1^2 \quad (6)$$

$$k_2 = a^2 + R^2 - R_2^2 \quad (7)$$

μ_1, μ_2 = anomalies of the satellite at the moment of the first and second range measurements respectively.

R_1, R_2 = ranges from ship to satellite at the time of the first and measurements respectively.

R = radius of the earth at the ship position.

a = radius of the satellite orbit.

From the general law of error propagation for random and uncorrelated errors.

$$\sigma_\nu^2 = \left(\frac{\partial \nu}{\partial R_1} \right)^2 \cdot \sigma_{R_1}^2 + \left(\frac{\partial \nu}{\partial R_2} \right)^2 \cdot \sigma_{R_2}^2 + \left(\frac{\partial \nu}{\partial R} \right)^2 \cdot \sigma_R^2 + \left(\frac{\partial \nu}{\partial a} \right)^2 \cdot \sigma_a^2 + \left(\frac{\partial \nu}{\partial \mu_1} \right)^2 \cdot \sigma_{\mu_1}^2 + \left(\frac{\partial \nu}{\partial \mu_2} \right)^2 \cdot \sigma_{\mu_2}^2 \quad (8)$$

$$\sigma_\mu^2 = \left(\frac{\partial \mu}{\partial R_1} \right)^2 \cdot \sigma_{R_1}^2 + \left(\frac{\partial \mu}{\partial R_2} \right)^2 \cdot \sigma_{R_2}^2 + \left(\frac{\partial \mu}{\partial R} \right)^2 \cdot \sigma_R^2 + \left(\frac{\partial \mu}{\partial a} \right)^2 \cdot \sigma_a^2 + \left(\frac{\partial \mu}{\partial \mu_1} \right)^2 \cdot \sigma_{\mu_1}^2 + \left(\frac{\partial \mu}{\partial \mu_2} \right)^2 \cdot \sigma_{\mu_2}^2 \quad (9)$$

The expressions of the partial derivatives are:

$$\left\{ \begin{aligned}
 \frac{\partial \nu}{\partial R_1} &= \frac{R_1 \cdot \sin (\mu_2 - \mu)}{a \cdot \sin \nu \cdot \sin (\mu_2 - \mu_1)} \\
 \frac{\partial \nu}{\partial R_2} &= \frac{R_2 \cdot \sin (\mu_2 - \mu)}{a \cdot \sin \nu \cdot \sin (\mu_2 - \mu_1)} \\
 \frac{\partial \nu}{\partial R} &= \frac{a \cdot \sin (\mu_2 - \mu_1) \cos \nu - R [\sin (\mu - \mu_1) + \sin (\mu_2 - \mu)]}{a \sin \nu \sin (\mu_2 - \mu_1)} \\
 \frac{\partial \nu}{\partial a} &= \frac{R \cdot \sin (\mu_2 - \mu_1) \cdot \cos \nu - a [\sin (\mu - \mu_1) + \sin (\mu_2 - \mu_1)]}{a \sin \nu \sin (\mu_2 - \mu_1)} \\
 \frac{\partial \nu}{\partial \mu_1} &= \frac{\sin (\mu - \mu_1) \sin (\mu_2 - \mu) \cos \nu}{\sin \nu \sin (\mu_2 - \mu_1)} \\
 \frac{\partial \nu}{\partial \mu_2} &= \frac{-\sin (\mu - \mu_1) \sin (\mu_2 - \mu) \cos \nu}{\sin \nu \cdot \sin (\mu_2 - \mu_1)} \\
 \frac{\partial \mu}{\partial R_1} &= \frac{R_1 \cdot \cos (\mu_2 - \mu)}{a \sin (\mu_2 - \mu_1)} \\
 \frac{\partial \mu}{\partial R_2} &= \frac{-R_2 \cdot \cos (\mu - \mu_1)}{a \cdot \sin (\mu_2 - \mu_1)} \\
 \frac{\partial \mu}{\partial R} &= \frac{R \cdot [\cos (\mu - \mu_1) - \cos (\mu_2 - \mu)]}{a \sin (\mu_2 - \mu_1)} \\
 \frac{\partial \mu}{\partial a} &= \frac{\cos (\mu - \mu_1) \cdot \cos (\mu_2 - \mu)}{\sin (\mu_2 - \mu_1)} \\
 \frac{\partial \mu}{\partial \mu_1} &= \frac{\cos (\mu - \mu_1) \cdot \cos (\mu_2 - \mu)}{\sin (\mu_2 - \mu_1)} \\
 \frac{\partial \mu}{\partial \mu_2} &= \frac{\cos (\mu_2 - \mu) \cos (\mu - \mu_1)}{\sin (\mu_2 - \mu_1)}
 \end{aligned} \right. \quad (10)$$

$$\left\{ \begin{aligned}
 \frac{\partial \mu}{\partial R_1} &= \frac{R_1 \cdot \cos (\mu_2 - \mu)}{a \sin (\mu_2 - \mu_1)} \\
 \frac{\partial \mu}{\partial R_2} &= \frac{-R_2 \cdot \cos (\mu - \mu_1)}{a \cdot \sin (\mu_2 - \mu_1)} \\
 \frac{\partial \mu}{\partial R} &= \frac{R \cdot [\cos (\mu - \mu_1) - \cos (\mu_2 - \mu)]}{a \sin (\mu_2 - \mu_1)} \\
 \frac{\partial \mu}{\partial a} &= \frac{\cos (\mu - \mu_1) \cdot \cos (\mu_2 - \mu)}{\sin (\mu_2 - \mu_1)} \\
 \frac{\partial \mu}{\partial \mu_1} &= \frac{\cos (\mu - \mu_1) \cdot \cos (\mu_2 - \mu)}{\sin (\mu_2 - \mu_1)} \\
 \frac{\partial \mu}{\partial \mu_2} &= \frac{\cos (\mu_2 - \mu) \cos (\mu - \mu_1)}{\sin (\mu_2 - \mu_1)}
 \end{aligned} \right. \quad (11)$$

It can be noted that in all of Equation 10 the term $\sin \nu$ appears in the denominator. Thus, these sensitivity coefficients which determine the variance of ν become large when the ship-point is near the orbital plane.

Furthermore, because $\sin(\mu_2 - \mu_1)$ appears in the denominator of all partial derivatives, the two measurements are to be taken as far apart as possible for best precision (measurements at the lowest possible elevation angle).

A sample computation for a typical case follows. It is assumed that a satellite pass takes place, for which $\nu = 11^\circ$. The origin of the anomalies is taken as the satellite position at the first range measurement. Therefore, the anomaly between the two measurements is at most (figure 2) $\mu_2 - \mu_1 = 21^\circ$. These figures are consistent with what can be expected from the GEOS B orbit. The ship position anomaly is symmetrical with respect to μ_1 and μ_2 . Thus

$$\mu_1 = 0^\circ \quad \mu = 10.5^\circ \quad \mu_2 = 21^\circ \quad \nu = 11^\circ$$

$$R_1 \approx R_2 \approx 1.2 \cdot 10^6 \text{ m} \quad R \approx 6.37 \cdot 10^6 \text{ m} \quad a = 7.37 \cdot 10^6 \text{ m}$$

The following error figures are assumed, conservatively consistent with the state of the art in the GEOS-B time frame.

$$\sigma_{R_1} = \sigma_{R_2} = \pm 9 \text{ m} \quad \Delta R_1 = \Delta R_2 = \pm 18 \text{ m}$$

$$\sigma_a = \pm 100 \text{ m} \quad \sigma_R = \pm 50 \text{ m} \quad \sigma_{\mu_1} = \sigma_{\mu_2} = \pm 100 \text{ m}$$

Where the σ indicate random errors and Δ indicate bias errors.

From equation 7 and 8 follows:

$$\frac{\partial \nu}{\partial R_1} = \frac{\partial \nu}{\partial R_2} = 0.4338 \quad \frac{\partial \nu}{\partial R} = 0.5369 \quad \frac{\partial \nu}{\partial a} = -0.8822$$

$$\frac{\partial \nu}{\partial \mu_1} = -\frac{\partial \nu}{\partial \mu_2} = 0.4765 \quad \frac{\partial \mu}{\partial R_1} = -\frac{\partial \mu}{\partial R_2} = 0.4467$$

$$\frac{\partial \mu}{\partial R} = \frac{\partial \mu}{\partial a} = 0 \quad \frac{\partial \mu}{\partial \mu_1} = \frac{\partial \mu}{\partial \mu_2} = 0.4999$$

$$\frac{\partial \nu}{\partial R_1} \cdot \sigma_{R_1} = \frac{\partial \nu}{\partial R_2} \cdot \sigma_{R_2} = 3.9 \text{ m} \quad \frac{\partial \nu}{\partial R} \cdot \sigma_R = 26.8 \text{ m} \quad \frac{\partial \nu}{\partial a} \cdot \sigma_a = 88.2 \text{ m}$$

$$\frac{\partial \nu}{\partial \mu_1} \cdot \sigma_{\mu_1} = - \frac{\partial \nu}{\partial \mu_2} \cdot \sigma_{\mu_2} = 47.6 \text{ m}$$

$$\frac{\partial \nu}{\partial R_1} \cdot \Delta R_1 = \frac{\partial \nu}{\partial R_2} \cdot \Delta R_2 = 7.8 \text{ m}$$

$$\sigma_\nu = \pm 114.3 \text{ m} \quad \Delta_\nu = \pm 15.6 \text{ m}$$

$$\frac{\partial \mu}{\partial R_1} \cdot \sigma_{R_1} = - \frac{\partial \mu}{\partial R_2} \cdot \sigma_{R_2} = 4.0 \text{ m} \quad \frac{\partial \mu}{\partial \mu_1} \sigma_{\mu_1} = \frac{\partial \mu}{\partial \mu_2} \cdot \sigma_{\mu_2} = 50.0 \text{ m}$$

$$\sigma_\mu = \pm 70.8 \text{ m} \quad \Delta_\mu = 0$$

Where, as usual, the σ denote random errors and the Δ denote bias errors.

It is to be noted that the errors are well below the requirements of the Apollo ship program.

It can be seen, also, that the terms deriving from satellite range inaccuracies are not predominant. Therefore, an improvement in the precision of the method is mainly dependent on improvement of the satellite ephemerides. (If the assumption $\sigma_\mu = \sigma_{\mu_1} = \sigma_{\mu_2} = \pm 50 \text{ m}$ can be held, the results would be: $\sigma_\nu = \pm 61.9 \text{ m}$ $\sigma_\mu = 35.6 \text{ m}$)

Assuming that the two components of position error are independent (a normally conservative assumption) 90.7m is obtained for the radius of the Circle of Probable Error (CPE). The Apollo ship program specifications are CPE = 300m. The position fix obtained with the present method can be expected to satisfy this specification 99.95% of the time. This conclusion is valid, however, only as the basic hypotheses are valid. The hypotheses concerning the measurement and ephemerides error levels are believed to be conservative for the GEOS B time frame. The problem geometry, too, is believed to be fairly typical. In fact, an increase in the great circle distance ν , will cause the allowable anomalies difference $(\mu_2 - \mu_1)$ to decrease. Therefore, it can be seen from equation 7 that a kind of compensation in the parameter sensitivity coefficient will take place. Nevertheless, ship position fixes where $\nu < 5^\circ$ are to be avoided (Reference 9).

APPENDIX C

PROBABILITY OF SATELLITE ACQUISITION

Probability of satellite acquisition is a dimensionless quantity defined as

$$P_1 = \frac{t}{T} \quad (1)$$

where t is the time the satellite is within the circle of view during the total period of time T . The period of time T must be long enough to insure several passes across the circle of view. Circle of view is the sector of sky wherein the satellite must be to be tracked by the ground station. For a navigation satellite, this sector is actually a circle.

If a rigorous analytical approach to the computation of P_1 is to be attempted, the following relationship (reference 2) can be used:

$$P_1 = \int_{L_1}^{L_2} \frac{\Delta\varphi \cdot \cos L \cdot dL}{2\pi^2 \sqrt{\sin^2 i - \sin^2 L}} \quad (2)$$

This relationship is valid for circular orbits and for satellite ascending nodes uniformly distributed around the equator. The latter condition is reasonably satisfied if long periods of time are considered.

The meaning of the symbols are:

P_1 is the probability of acquiring the satellite,

L_1 and L_2 are the lowest and highest latitudes in the boundary of the circle of view from the ship. Circle of view is defined as the intersection of the cone whose apex is at the ship position and whose semi-aperture is $(90^\circ - \alpha)$ and the sphere whose radius is equal to the satellite distance from the center of the earth. If the satellite position is inside the circle of view, useful measurements can be made. In the case where the circle extends to latitudes above $+i$ or below $-i$, the limits become $+i$ and $-i$ respectively. When $L = i$ the integrand becomes infinite; nevertheless, the integral is still finite (reference 2, Appendix B).

$\Delta\varphi$ is the change in longitude across the circle of view at the general latitude L .

i is the inclination angle of the orbital plane.

For the case under examination, the quantity $\Delta\varphi$ is analytically

expressible:

$$\Delta\varphi = 2 \arccos \left(\frac{\sin L_o}{\cos L \cos L_p} - \tan L_p \tan L \right) \quad (3)$$

$$L_o = \alpha + \arcsin \left(\frac{R \cos \alpha}{R + H} \right) \quad (4)$$

where:

L_p = latitude of the ship

R = mean earth's radius

H = satellite altitude

α = minimum antenna elevation angle.

Two approaches are available for the numerical integration of the equation. The first one consists of expanding the integrand into series in L and integrating term by term. In this case the expression for P will have the form of a polynomial in L_1 and L_2 . In order to have formulae of some unity, the series expansions are to be truncated at a certain degree of L , according to the desired accuracy. The arc cosine series, however, is slowly convergent when L approaches unity ($L \rightarrow 57^\circ$). In this case, several terms of the series must be retained for an acceptable accuracy. Therefore, the practical validity of this approach is questionable if the ships are not restricted to the equatorial zone.

The general approach to the solution is to perform the numerical integration by means of a method that would fit particularly well to the problem under examination. If a diagram like figure 1 is to be derived, computations for several values of the ship latitude L_p are to be made. Therefore, a self-starting step-by-step method of integration to be used in connection with a digital computer is appealing. In the Runge-Kutta method, each integration step is made without the use of values of the function preceding the present step. Therefore, no special methods are required to start the computation. Additionally, the integration interval can be changed at any step whatsoever of the integration. The fundamental idea involved in this method is to obtain an expression for the integral function after a step of integration, which is accurate to terms of a

a certain order in $h = dL$, without using derivatives of the integrand in the computation. Four substitutions into the differential equation are required to obtain an approximation to the fourth order in h . Variations of the method well suited to digital computer techniques, allow change in the value of h during computation as a function of the desired accuracy.